Kittel TP.

4.1 (Photon excitations are taken to be interpreted as the # of photons, quantum # s $\cong$ s photons.)

In thermostatistics, we can safely take expectation value of the quantum # as "the quantum #" that will be observed, allowing us to safely take

$$N = \sum_n D(s_n)\, s_n = \sum D(s_n) \langle s_n \rangle$$

where $D(s_n)$ is degeneracy, s_n is the quantum #.

Using planck distribution function for $\langle s_n \rangle$, and 3D particle in a box degeneracy formula for $D(s_n)$ we have

$$N = \frac{1}{8}(2) \int_0^\infty 4\pi n^2 \, dn \frac{1}{\exp\left(\frac{n\pi c}{L\tau}\right) - 1}, \quad \text{where } \omega_n = \frac{n\pi c}{L}.$$

$$= \pi \int_0^\infty dn \, n^2 \frac{1}{\exp\left[\frac{\hbar c n \pi}{L\tau}\right] - 1}$$

Introduce $x = \left(\frac{\hbar c \pi}{L\tau}\right) n$, then $n^2 = \left(\frac{L\tau}{\hbar c \pi}\right)^2 x^2$, $dn = \left(\frac{L\tau}{\hbar c \pi}\right) dx$,

$$\Rightarrow \boxed{N = \pi \left(\frac{L\tau}{\hbar c \pi}\right)^3 \int_0^\infty dx \frac{x^2}{\exp(x) - 1}.}$$

The $\int_0^\infty dx \frac{x^2}{\exp(x)-1}$ can be evaluated to give ≈ 2.40.

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